A Probability Based Approach to Estimating Costs of Capital for Small Business

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ABSTRACT. While discount rates of listed companies can be readily estimated using "betas" and the Capital Asset Pricing Model, the same is not true for small business. Entrepreneurs often have to rely on subjective assessments of the financial viability of their business ventures. This paper suggests an alternative to estimate the costs of capital for small businesses. Costs of capital are derived from the probability of success for similar business. These required rates of return can be used as minimum hurdle rates to assess the viability and profitability of the business under consideration. Since risk neutrality is assumed of investors in this approach, the costs of capital established should only be regarded as minimum returns required by risk-averse investors. Therefore, this suggested approach attempts to provide a refined "rule-of-thumb" which may be of value to small business entrepreneurs and financiers, especially when detailed accounting and financial data of similar business are not readily available.

1. Background

The discounted cash flows methodology is the standard valuation approach in Finance. The value of an asset is calculated as the present value of all future net cash flows. The capitalization rate required to discount future cash flows can be obtained using the Capital Asset Pricing Model (CAPM). This approach is workable as long as an appropriate measure of systematic risk (or "beta") is available so that CAPM can be applied.

Finding an appropriate beta is an easier task for publicly listed companies than it is for private small businesses. First, there is a lack of information on market values of small businesses. Second, a market index which includes values of small businesses is unavailable. Both pieces of information are required in estimating betas for small business ventures. Although the beta of similar projects undertaken by publicly listed firms sometimes may be used as a substitute, the size of most entrepreneurial business ventures is too small to make such an approach practicable. Although the CAPM approach may be relevant for a venture capital fund which holds a diversified portfolio of small businesses, it may not be appropriate from the point of view of the entrepreneur. The entrepreneur is likely to be the sole investor who has a high proportion of capital tied up in the business. Diversification may simply be unattainable in that case. The entrepreneur has to bear both unique and market risks, and needs to be rewarded accordingly. These are some of the reasons why further research is necessary to find the appropriate discount rates for small business ventures.

This paper derives probability based estimates for minimum costs of capital for small businesses. Section 2 looks at the probability of survival of small businesses. Section 3 translates the probability of survival into a cost of equity capital. The cost of debt is then derived in Section 4. Section 5 extends the analysis to a multi-period setting. A summary of results follows in Section 6.

2. Probability of survival of small businesses

Default risk accounts for a significant portion of risks borne by debt-holders. On the other hand, entrepreneurs and equity investors assume the residual risk which depends on, among others, the nature of the undertaking, the macroeconomic environment, and the managerial skills. The outcome of a business venture can be defined in

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terms of two mutually exclusive and exhaustive events – survival and failure. For the purpose of this paper, a business venture is defined to have failed when it could not meet its debt payments. When that happens, the venture is liquidated and the debt-holders recover part of their investment. Otherwise the venture is defined to have survived.

Suppose p is the probability of survival in any given year, so that $1 - p$ is the probability of failure. The probability of survival over a period of n years will then be p^n, and the corresponding probability of failure is $1 - p^n$, assuming p remains constant over the years.[1]

To illustrate the above, suppose 40% of all ventures in the bio-technology industry have failed over the last ten years. If the 40% actual failure rate is accepted as the best estimate of the probability of failure over the ten-year period, the probability of survival, p, is given by:

$$1 - p^{10} = 0.40,$$

$$p = 0.95$$

In this case, the probability of survival of a bio-technology business in any given year is 0.95. Since this estimate is derived from an industry average over a period of time, it may need to be adjusted to accommodate unique circumstances of the individual business. It should be noted that the probability of failure in a given year can be very different from the probability of failure over a period of time. Although the failure rate of some new business ventures can be high over a period of several years, the risk of failing in any given year could be substantially lower. Some investors are willing to accept the risk of failure in exchange for high expected returns.

3. Cost of equity for small ventures

Bierman et al. (1975, 1990) developed a model to estimate risk-adjusted discount rates for junk bonds. A similar approach is used in the following to estimate a risk-adjusted rate for the equity of small business ventures. Assume that

- investors are risk-neutral;
- p is the probability of survival per period;
- entrepreneur/equity investors receive nothing in the case of default;
- there is no tax.

Moreover, it can be assumed that the venture is financed by D dollars in debt and E dollars in equity, although these two parameters disappear in the expressions for costs of capital.[2]

Consider the one-period case:

$$p \times [(D + E) \times (1 + r_a) - D \times (1 + r_d)] + (1 - p) \times 0 = E \times (1 + i) \tag{1}$$

$$p \times [D \times (1 + r_d) + E \times (1 + r_e) - D \times (1 + r_d)] + (1 - p) \times 0 = E \times (1 + i) \tag{2}$$

$$(1 + r_e) = \frac{1 + i}{p}$$

$$r_e = \frac{1 + i}{p} - 1 = \frac{i}{p} + \frac{1 - p}{p} \tag{3}$$

$$r_e - i = \frac{i}{p} - i + \frac{1 - p}{p} = (1 + i) \times \frac{1 - p}{p} \tag{4}$$

where r_a, r_d and r_e are the costs of capital, cost of debt and cost of equity respectively, and i represents the risk-free rate of interest.

Under the risk-neutral assumption, the expected return for all risky assets is equal to the risk-free rate of return in equilibrium. The expected cash flow that belongs to equity-holders is $p \times [(D + E) \times (1 + r_a) - D \times (1 + r_d)]$, and this is equated to the equity invested plus the risk-free rate of return on equity $E \times (1 + i)$ in Equation (1). Equation (2) follows from the definition of the weighted average cost of capital, $(1 + r_a) = [D/(D + E)] \times (1 + r_d) + [E/(D + E)] \times (1 + r_e)$. Equation (3) gives the cost of equity, while Equation (4) shows the risk premium for equity investors, which is always positive when the probability of failure is between zero and one.

4. Estimating cost of debt

The following analysis assumes that

- debt investors are risk-neutral;
- p is the probability of survival each period;
- investors receive a rate of return r_b which is lower than the risk-free rate in case of default.[3]

Consider the one-period case,

$$p \times D \times (1 + r_d) + (1 - p) \times D \times (1 + r_b) = D \times (1 + i); \qquad -1 \le r_b \le i$$

$$r_d = \frac{i - r_b}{p} + r_b = \frac{i}{p} - \frac{1 - p}{p} \times r_b \tag{5}$$

$$r_d - i = \frac{i}{p} - i - \frac{1 - p}{p} \times r_b = (i - r_b) \times \frac{1 - p}{p} \tag{6}$$

where r_d is the cost of debt and i is the risk-free rate.

The expression $D \times [p(1 + r_d) + (1 - p)(1 + r_b)]$ is simply the expected pay-off for the debt. This is equated to $D \times (1 + i)$, the pay-off for a risk-free debt, on the assumption that investors are risk-neutral. The cost of debt is given in Equation (5). Given that the "residue return" r_b is smaller than i, the risk premium, $r_d - i$, is positive as indicated in Equation (6). Equation (6) can also be interpreted as the difference between the risk-free return and the residue return multiplied by the odds of default. Note that this risk premium is a function of the risk-free rate, probability of default, and the residue return. Comparing Equations (6) and (4), it is clear that equity-holders require a higher rate of return than debt-holders as long as r_b is greater than negative one. When r_b equals negative one, debt-holders require the same risk premium since in this case they also receive nothing should the firm fail.[4]

5. Multi-period setting

The results developed in Section 3 also apply in a multi-period setting if the assumptions of constant p and no interim dividend pay-out are maintained. These two assumptions can be relaxed in the multi-period case. First, assume that equity-holders receive no dividends until the business venture is sold, which is a reasonable assumption for small business ventures. The special case when the probability of survival is not constant over time is considered. Suppose the probability of survival is p_1 in periods one to $n - 1$, when no dividend is paid, and p_2 in period n, when the business is sold. In period n,

$$p_2 \times \{p_1^{n-1}(1 + r_e)^n\} + (1 - p_2) \times (0) = (1 + i)^n$$

$$r_e = (1 + i) \times p_2^{-1/n} \times p_1^{(1-n)/n} - 1 \tag{7}$$

If $p_1 = p_2 = p$, Equation (7) becomes

$$r_e = \frac{1 + i}{p} - 1,$$

which is equivalent to equation (3).

The results can also be modified if there are interim cash flows. Assume an interest-only debt which matures in n periods, and for simplicity the probability of default is constant over the n periods of time. Again assume that when the firm defaults, debt-holders recover \$$(1 + r_b)$ per dollar invested, where $-1 \le r_b \le i$. The following figure illustrates the situation:

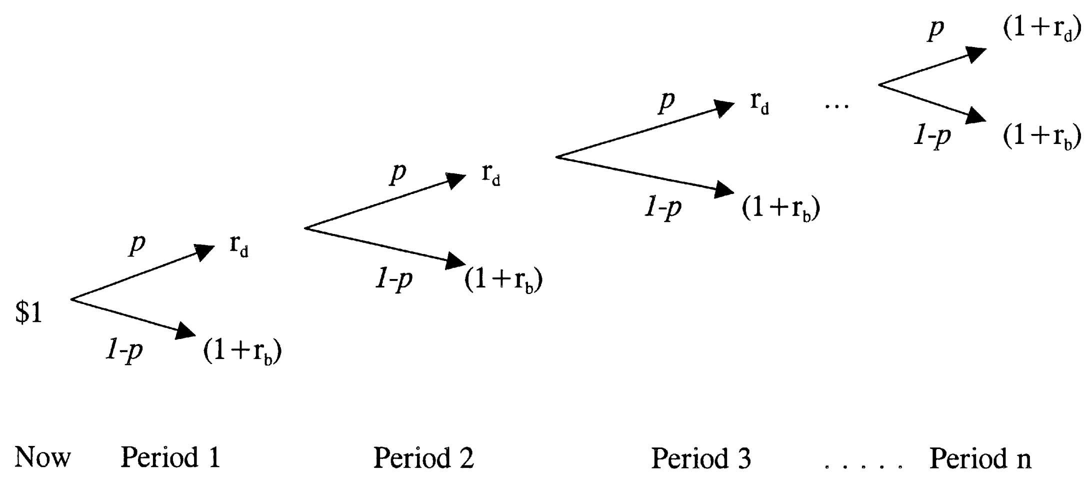

Figure 1. *N*-period interest-only loan.

The expected pay-off at time n to the debt-holders is given by (see Appendix):

$$p^n + (1 - p^n) \times \left[r_d \times \frac{p}{1 - p} + (1 + r_b) \right] \quad (8)$$

Under the risk-neutrality assumption, Equation (8) can be equated to a similar *n*-period risk-free debt, where there is zero probability of default (i.e. $1 - p = 0$):

$$p^n + (1 - p^n) \times \left[r_d \times \frac{p}{1 - p} + (1 + r_b) \right] = 1 + n \times i$$

$$r_d = \left[\frac{1 + n \times i - p^n}{1 - p^n} - (1 + r_d) \right] \times \frac{1 - p}{p} \quad (9)$$

If debt-holders receive nothing in case of default ($r_b = -1$), debt-holders would require the same return as equity-holders, and $r_d = r_e$. Therefore, if a dividend policy of paying r_e per period is followed, and under the simplifying assumption that the dividend amount r_e is always paid as long as the business survives, the cost of equity becomes:[5]

$$r_e = \frac{1 + n \times i - p^n}{1 - p^n} \times \frac{1 - p}{p} \quad (10)$$

Given p, i, r_b and n, r_d and r_e can be readily determined. Note that when n equals one, Equation (9) is reduced to Equation (5), while Equation (10) is reduced to Equation (3). Other assumptions regarding the dividends policy and debt structure in the multi-period setting can also be examined and used to derive the "risk-neutral" costs of capital.

Table I contains some hypothetical values to illustrate Equations (3) and (5) in the one-period case, while Table II provides an illustration of Equations (9) and (10) in the multi-period case.

6. Summary

This paper provides estimates of the minimum required costs of capital for small business ventures using a probability based model. Although the equations derived for the cost of debt and cost of equity are simple to apply, they should be regarded as some "base-line" estimates only because of the risk-neutrality assumption. The main advantage of the probability based approach

TABLE I
Hypothetical values to calculate r_d and r_e – one period case
Assume i = 10% and r_b = –50%

Annual probability of survival	$r_e \left(= \frac{i}{p} + \frac{1-p}{p} \right)$	$r_d \left[= \left(\frac{i}{p} - r_b \times \frac{1-p}{p} \right) \right]$
1.00	10.00%	10.00%
0.95	15.79%	13.16%
0.90	22.22%	16.67%
0.85	29.41%	20.59%
0.80	37.50%	25.00%
0.75	46.67%	30.00%
0.70	57.14%	37.51%
0.60	83.33%	50.00%
0.50	120.00%	70.00%

TABLE II
Hypothetical values to calculate r_d and r_e – multi-period case
Assume i = 10%, r_b = –50%, and n = 5

Annual probability of survival	$r_e \left[= \left(\frac{1 + in - p^n}{1 - p^n} \right) \times \frac{1-p}{p} \right]$	$r_d \left\{ = \left[\frac{1 + in - p^n}{1 - p^n} - (1 + r_b) \right] \times \frac{1-p}{p} \right\}$
1.00	10.00%	10.00%
0.95	16.90%	14.26%
0.90	24.68%	19.12%
0.85	33.51%	24.68%
0.80	43.59%	31.09%
0.75	55.19%	38.52%
0.70	68.61%	47.19%
0.60	102.81%	69.48%
0.50	151.61%	101.61%

is that it does not require historical estimates of market risk using market values of debt and equity of similar business ventures. These market value data are often not available. Instead, an estimate of the probability of default is required. The information required to estimate default rate is usually in the public domain. The minimal amount of data required should provide practitioners such as venture capitalists and bank loan officers with a quick rule-of-thumb to estimate the minimum required returns for small business projects. The analysis in this paper can be extended to cover other loan structures and dividends policy. Further research can improve the precision of the estimates and widen the range of applications.

Appendix

Proposition

The expected value of the total end-of-period cash flows of the interest-only debt is given by Equation (8):

$$p^n + (1 - p^n) \times \left[r_d \times \frac{p}{1-p} + (1 + r_b) \right]$$

This can be proved by mathematical induction.

Step 1: n = 1

In this case, the end-of-period cash flow is $(1 + r_d)$ with prob. p, or $(1 + r_b)$ with prob. $(1 - p)$.

Therefore, expected end-of-period cash flow = $(1 + r_d) \times p + (1 + r_b) \times (1 - p)$.

Setting $n = 1$ in (8):

$$p + (1 - p) \times \left[r_d \times \frac{p}{1-p} + (1 + r_b) \right]$$
$$= (1 + r_d) \times p + (1 + r_b) \times (1 - p)$$

The proposition is true for $n = 1$.

Step 2: Assume the proposition is true for n = k. To show that it is also true for n = k + 1.

Assume expected total cash flow with k periods = $p^k + (1 - p^k) \times [r_d \times p/(1 - p) + (1 + r_b)]$

With an extra period to go, the \$1 initial investment will not be paid at the end of period k, so the expected total cash flow at end of period $k = (1 - p^k) \times [r_d \times p/(1 - p) + (1 + r_d)]$.

Let ECF$(k + 1)$ be the expected total cash flow at the end of the $(k + 1)$th period.

$$\begin{aligned}
&\text{ECF}(k+1) \\
&= p \times \left\{ (1 - p^k) \times \left[r_d \times \frac{p}{1-p} + (1 + r_b) \right] + r_d \right\} \\
&\quad + p^{k+1} \times 1 + (1 - p) \times (1 + r_b) \qquad (11) \\
&= p^{k+1} + p \times (1 - p^k) \times \left[r_d \times \frac{p}{1-p} + (1 + r_b) \right] \\
&\quad + p \times r_d + (1 - p) \times (1 + r_b) \\
&= p^{k+1} + p \times (1 - p^k) \times \left[r_d \times \frac{p}{1-p} + (1 + r_b) \right] \\
&\quad + (1 - p) \times \left[r_d \times \frac{p}{1-p} + (1 + r_b) \right] \\
&= p^{k+1} + [p \times (1 - p^k) + (1 - p)] \\
&\quad \times \left[r_d \times \frac{p}{1-p} + (1 + r_b) \right] \qquad (12) \\
&= p^{k+1} + (1 - p^{k+1}) \times \left[r_d \times \frac{p}{1-p} + (1 + r_b) \right] \qquad (13)
\end{aligned}$$

The second term in Equation (11), $p^{k+1} \times 1$, is the probability of getting the initial investment back after $k + 1$ period times the amount invested, i.e. the expected value of the \$1 invested to be received $k + 1$ periods later. The fact $p \times (1 - p^k) + (1 - p) = p \times (1 - p) \times (1 + p + \ldots + p^{k-1}) + (1 - p) = (1 - p) \times (1 + p + \ldots + p^k) = 1 - p^{k+1}$ is used in going from Equations (12) to (13).

Step 3: The proposition is true for all n by the principle of mathematical induction.

Notes

1 The assumption that p remains constant is unrealistic. It is more likely that p changes throughout the life of the venture. This case is used for illustration purposes only, before the general case where p can change over time is discussed in Section 5.

2 This does not imply that the costs of capital are independent of capital structure. Under the framework discussed, any leverage effect is captured by the probability of failure $(1 - p)$ which is higher when the firm has more debt, and the recovery rate r_b for debt-holders, which decreases with higher leverage.

3 Clearly, r_b cannot be less than –1. When r_b equals –1, debt holders receive nothing when the firm fails. Also, under the risk-neutral assumption, the case of r_b being greater than the risk-free rate need not be considered since it implies that investing in the debt of the firm is risk-free and the return is higher than the risk-free rate.

4 r_b can be a "choice variable" from the point of view of the lender. The lender can to a certain extent control the ex-ante expected residue risk by specifying safeguard measures such as the use of collateral, sinking funds requirements and other monitoring devices.

5 The company may pay equity-holders less than r_e or skip dividends altogether, without triggering default. These possibilities are not considered in this paper for simplicity. Therefore, the cost of equity derived under the assumption of a constant dividend r_e per period will again represents the

minimum required rate of return for equity-holders who in fact may not receive the specified dividend payments.

References

Bierman, H. and J. E. Hass, 1975, 'An Analytical Model of Bond Risk Differentials', *Journal of Financial and Quantitative Analysis* (December), 757–773.

Bierman, H., 1990, 'Investing in Junk Bonds', *Journal of Portfolio Management* (Winter), 60–62.

Yawitz, Jess B., 1977, 'An Analytical Model of Interest Rate Differentials and Different Default Recoveries', *Journal of Financial and Quantitative Analysis* (September), 481–490.

Yawitz, Jess B., Kevin J. Maloney and Louis H. Ederington, 1985, 'Taxes, Default Risk, and Yield Spreads', *Journal of Finance* (September), 1127–1140.